Discrete Mathematics

In this technological age, mathematics is more important than ever. When students leave school, they are more and more likely to use mathematics in their work and everyday lives — operating computer equipment, planning timelines and schedules, reading and interpreting data, comparing prices, managing personal finances, and completing other problem-solving tasks. What they learn in mathematics and how they learn it will provide an excellent preparation for a challenging and ever-changing future.

The state of Indiana has established the following mathematics standards to make clear to teachers, students, and parents what knowledge, understanding, and skills students should acquire in Discrete Mathematics:

Standard 1 — Counting Techniques

Students develop an understanding of combinatorial reasoning, using various types of diagrams and the fundamental counting principle to find numbers of outcomes and related probabilities. They also use simulations to solve counting and probability problems.

Standard 2 — Matrices

Students understand how matrices can be used to store and organize data and to solve systems of equations. They also use matrices to solve Markov chain problems that link present events to future events using probabilities.

Standard 3 — Recursion

Students understand and apply recursive methods to solve problems, including the use of finite differences.

Standard 4 — Graph Theory

Students understand how graphs of points joined by lines can model a variety of problem situations. These include critical path analysis, graph coloring problems, minimal spanning trees, and bin-packing techniques.

Standard 5 — Social Choice

Students analyze election data to evaluate different election methods and use weighted voting techniques to decide voting power within a group. They understand and use fair division techniques to solve apportionment problems.

Standard 6 — Linear Programming

Students understand how to use diagrams to solve simple optimization problems and extend this to the Simplex method for solving more general optimization problems.

Standard 7 — Game Theory

Students understand and use game theory methods to solve strictly determined games and nonstrictly determined games.

As part of their instruction and assessment, students should also develop the following learning skills by Grade 12 that are woven throughout the mathematics standards:

Mathematical Reasoning and Problem Solving

In a general sense, mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to combinatorial reasoning, recursive thinking, critical path analysis, and other counting situations.

Communication

The ability to read, write, listen, ask questions, think, and communicate about math will develop and deepen students' understanding of mathematical concepts. Students should read text, data, tables, and graphs with comprehension and understanding. Their writing should be detailed and coherent, and they should use correct mathematical vocabulary. Students should write to explain answers, justify mathematical reasoning, and describe problem-solving strategies.

Representation

The language of mathematics is expressed in words, symbols, formulas, equations, graphs, and data displays. The concept of one-fourth may be described as a quarter, ¼, one divided by four, 0.25, ⅛ + ⅛, 25 percent, or an appropriately shaded portion of a pie graph. Higher-level mathematics involves the use of more powerful representations: exponents, logarithms, π, unknowns, statistical representation, algebraic and geometric expressions. Mathematical operations are expressed as representations: +, =, divide, square. Representations are dynamic tools for solving problems and communicating and expressing mathematical ideas and concepts.

Connections

Connecting mathematical concepts includes linking new ideas to related ideas learned previously, helping students to see mathematics as a unified body of knowledge whose concepts build upon each other. Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas (algebra, geometry, the entire number system). Mathematics is also the common language of many other disciplines (science, technology, finance, social science, geography) and students should learn mathematical concepts used in those disciplines. Finally, students should connect their mathematical learning to appropriate real-world contexts.

Standard 1

Counting Techniques

Students use counting techniques.

DM.1.1 Use networks, traceable paths, tree diagrams, Venn diagrams, and other pictorial representations to find the number of outcomes in a problem situation.

Example: In a motel there are 4 different elevators that go from Joan's room to the pool and 3 different doors to the pool area. Use a tree diagram to show how many different ways Joan can get from her room to the pool.

DM.1.2 Use the fundamental counting principle to find the number of outcomes in a problem situation.

Example: You are getting dressed one morning when you realize that you have far too many choices. You have 6 shirts to choose from, 4 pairs of jeans, and 3 pairs of shoes. Ignoring color coordination, construct a tree diagram or other pictorial representation to show how many different outfits you could assemble.

DM.1.3 Use combinatorial reasoning to solve problems.

Example: You know that your locker combination contains the numbers 2, 4, 6, and 8, but you have forgotten the order in which they occur. What is the maximum number of combinations you need to try before your locker opens?

DM.1.4 Use counting techniques to solve probability problems.

Example: In the last example, what is the probability that your locker opens with the first combination you try?

DM.1.5 Use simulations to solve counting and probability problems.

Example: A panel of 12 jurors was selected from a large pool that was 70% male and 30% female. The jury turned out to be 11 men and 1 woman. Suspecting gender bias, the defense attorneys asked how likely is it that this situation, or worse, would occur purely by chance. Simulate this situation using a random number generator to select 12 numbers, letting 0, 1, and 2 represent women and 3, 4, 5, 6, 7, 8, and 9 represent men. Note the number of times that 11 or 12 men are chosen.

Standard 2

Matrices

Students use matrices.

DM.2.1 Use matrices to organize and store data.

Example: Central High School offers three different styles of class rings — standard, classic, and deluxe. Each style is available in a girl's ring and a boy's ring. Make up your own data to show how many of each variety was sold and store it in a matrix with rows and columns labeled.

DM.2.2 Use matrix operations to solve problems.

Example: Suppose the rings in the previous problem cost $90, $120, and $135 for the girls' rings and $110, $140, and $165 for the boys' rings. Display this information in a matrix and use matrix multiplication to find the total revenue from the sale of girls' rings and boys' rings.

DM.2.3 Use row-reduction techniques to solve problems.

Example: Solve this system of equations using an augmented matrix and row reduction:

$$\begin{aligned} x - 2y + 3z &= 5 \\ x \quad\quad + 3z &= 11 \\ 5y - 6z &= 9 \end{aligned}$$

DM.2.4 Use the inverse of a matrix to solve problems.

Example: Solve the system of equations in the last example using an inverse matrix.

DM.2.5 Use Markov chains to solve problems.

Example: If a student does homework one day, there is a 70% probability that he or she will do it again the next day. If a student does not do homework one day, there is a 60% probability that he or she will not do it again the next day. On Thursday, 75% of the students did their homework. What can you expect to happen on Friday?

DM

Standard 3

Recursion

Students use recursive techniques.

DM.3.1 Use recursive thinking to solve problems.

Example: How many handshakes would occur in this room if everyone shook hands with everyone else? Create a spreadsheet that will find the number of handshakes starting with one person and increasing the number to 15.

DM.3.2 Use finite differences to solve problems.

Example: Add two columns to the spreadsheet from the previous example and create appropriate formulas for each to calculate first and second differences.

Standard 4

Graph Theory

Students use graph theory techniques.

DM.4.1 Use graphs consisting of vertices and edges to model a problem situation.

Example: There are two islands in the River Seine in Paris. The city wants to construct four bridges that connect each island to each side of the riverbank and one bridge that connects the two islands directly. The city planners want to know if it is possible to start at one point, cross all five bridges, and end up at the same point without crossing a bridge twice. Use a graph to help solve this problem.

DM.4.2 Use critical path analysis to solve scheduling problems.

Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project.

DM.4.3 Use graph coloring techniques to solve problems.

Example: Color a map of the Midwestern states of the United States so that no adjacent states are the same color. What is the minimum number of colors needed?

DM.4.4 Use minimal spanning trees to solve problems.

Example: The telephone company wants to connect cities with new telephone lines. They calculate the cost of connecting each pair of cities, but want to reduce costs by connecting cities through others. Given a graph showing the cost of connecting each pair of cities, find the minimum cost for connecting all the cities with new telephone lines.

DM.4.5 Use bin-packing techniques to solve problems.

Example: Six large crates of electronic equipment are to be shipped to a warehouse. The crates weigh 2,800, 6,000, 5,400, 1,600, 6,800, and 5,000 pounds. Each delivery truck has a capacity of 10,000 pounds. What is the minimum number of trucks needed to send all the crates?

Standard 5

Social Choice

Students use the mathematics of social choice.

DM.5.1 Use election theory techniques to analyze election data.

Example: Each student in your class ranks four kinds of pop from the most preferred to least preferred. Discuss the merits of various methods for deciding on the overall ranking by the class.

DM.5.2 Use weighted voting techniques to decide voting power within a group.

Example: Company stockholders have different numbers of votes according to their holdings. For given holdings, find the power index of each stockholder.

DM.5.3 Use fair division techniques to divide continuous objects.

Example: Find a method for dividing a piece of cake among three people so that each person feels they have received a fair share.

DM.5.4 Use fair division techniques to solve apportionment problems.

Example: Find the enrollment of seniors, juniors, sophomores, and freshmen at your high school. If there are 20 seats on the Student Council, how should the representatives be apportioned so that the voting power of each class is proportional to its size?

Standard 6

Linear Programming

Students use linear programming techniques.

DM.6.1 Use geometric techniques to solve optimization problems.

Example: A company produces two varieties of widgets — standard and deluxe. A standard widget takes 3 hours to assemble and 6 hours to finish. A deluxe widget takes 5 hours to assemble and 5 hours to finish. The assemblers can work no more than 45 hours per week and the finishers can work no more than 60 hours per week. The profit is $32 on a standard widget and $40 on a deluxe widget. Use a graph to find how many of each model should be produced each week to maximize profit.

DM.6.2 Use the Simplex method to solve optimization problems with and without technology.

Example: Use the Simplex method to solve the problem in the last example.

Standard 7

Game Theory

Students use game theory.

DM.7.1 Use game theory to solve strictly determined games.

Example: Consider a card game where John gets a 4 of Hearts and a 5 of Clubs, and Susan gets a 3 of Clubs and a 6 of Hearts. The players each show one card simultaneously. The player who shows the card of larger value receives the sum of the numbers on the two cards shown. Set up the game matrix and find the optimal strategy and the value of the game.

DM.7.2 Use game theory to solve nonstrictly determined games.

Example: In the game "Two-Finger Morra," each of two players shows either one or two fingers. If the total number of fingers shown is even, Player A collects a dollar for each finger shown from Player B. If the total number of fingers is odd, Player A pays $3 to Player B. Set up the game matrix and find the optimal strategy and the value of the game.

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